

# CP Asymmetry In Neutral $B$ System At Symmetric Colliders\*

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## Abstract

Contrary to the conventional belief, time integrated asymmetry are measurable in selected final states in the neutral  $B$  system at symmetric  $e^+e^-$  colliders. They occur due to the interplay of weak and strong phases of two different amplitudes in addition to the  $B^0 - \bar{B}^0$  mixing. Observation of these asymmetries would be evidence for direct CP violation in the decay amplitudes.

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CP violation is one of the few remaining unresolved mysteries in particle physics. The explanation in the Standard Model based on Cabibbo-Kobayashi-Maskawa (CKM) matrix is still not established. Although there is no conflict between the observation of CP violation in the K-system and theory [1], intriguing hints of other plausible explanations emerge from astrophysical consideration of baryon to photon ratio in the universe [2]. Models based on additional Higgs bosons or gauge bosons can equally well explain the existing data [3]. It is for this reason that exploration of CP violation in the  $B$  system is so crucial. The  $B$  system offers several final states that provide a rich source for the study of this phenomena [4]. The principle techniques at electron-positron colliders that will be used involves: (a) measurement of particle-antiparticle partial rate asymmetries, and (b) rate asymmetries in the neutral  $B$  system with a lepton tag for one of the  $B$  mesons and the decay of the other  $B$  into a CP eigenstate (e.g.  $f - \psi K_S$ ) [5],

$$Asy = \frac{(l^+ f) - (l^- f)}{(l^+ f) + (l^- f)} . \quad (1)$$

The rationale for building an asymmetric electron-positron collider arises from the well known observation that time integrated asymmetries arising from  $B^0 - \bar{B}^0$  production in  $C = -1$  state are washed out due to quantum coherence of the initial state. On closer examination this is exactly correct only when the amplitude for the process  $B^0 \rightarrow f$  has a single weak phase. As we show below, when the amplitude is a mixture of two terms with different weak and strong phases, the asymmetry though diluted, still remains.

Consider the decay of the coherent  $B^0 \bar{B}^0$  state produced in  $C = -1$  state as in the decay of  $\Upsilon(4s)$ . If  $t_1$  and  $t_2$  denote decay times of the states that were pure  $B^0$  and  $\bar{B}^0$  at time zero, and  $f_a$  is a flavor specific decay like  $l^- \bar{\nu} D^*$  that tags pure  $\bar{B}^0$  state, while  $f_b$  is a CP eigenstate (e.g.  $\psi K_S, \pi^+ \pi^-, \pi^0 \pi^0 \dots$ ), then the rate is given by [5]

$$\begin{aligned} Rate(B^0(t_1)\bar{B}^0(t_2) \rightarrow f_a f_b) &\sim e^{-\Gamma(t_1+t_2)} \{ [1 + \cos(\Delta m(t_1 - t_2))] |A(f_b)\bar{A}(f_a)|^2 \\ &\quad + [1 - \cos(\Delta m(t_1 - t_2))] \left| \frac{q}{p} \right|^2 |\bar{A}(f_b)\bar{A}(f_a)|^2 \\ &\quad - 2 \sin \Delta m(t_1 - t_2) |A(f_b)\bar{A}(f_a)|^2 Im \left( \frac{q}{p} \frac{\bar{A}(f_b)}{A(f_b)} \right) \} \end{aligned} \quad (2)$$

where  $\Delta m$  is the mass difference between the heavier and lighter neutral  $B$  mesons,  $A$  and  $\bar{A}$  are amplitudes for  $B^0 \rightarrow f$  and  $\bar{B}^0 \rightarrow f$  respectively, and  $q$  and  $p$  are complex parameters defining  $B^0$  and  $\bar{B}^0$  mixing. The rate for  $B^0(t_1)\bar{B}^0(t_2) \rightarrow \bar{f}_a f_b$  is given by similar expression with replacement

$$\begin{aligned} A(f_b) &\rightarrow \bar{A}(f_b) , \\ \bar{A}(f_a) &\rightarrow A(\bar{f}_a) . \end{aligned} \tag{3}$$

For the leptonic mode which involves a single weak phase, we have

$$|\bar{A}(f_a)| = |A(\bar{f}_a)| . \tag{4}$$

In the  $B$  system  $|q/p| = 1$  to a very good approximation. If there is a single amplitude contributing to the decay  $B^0 \rightarrow f_b$ , we have  $|A(f_b)| = |\bar{A}(f_b)|$  and the cosine term drops off. The asymmetry then is proportional to  $\sin(\Delta m(t_1 - t_2))$ . This term vanishes when integrated over  $t_1, t_2$  from zero to infinity as would be the case for a symmetric collider. Consider now the case where the contribution to the decay  $B^0 \rightarrow f_b$  contains two contributions with different weak and strong phases. This situation arises when a process has both tree and penguin contributions. We can write in generality

$$A(B^0 \rightarrow f_b) = T e^{i(\delta_w + \delta_s)} + P , \tag{5}$$

where  $T$  and  $P$  stand for the tree and penguin contributions,  $\delta_w$  and  $\delta_s$  are the weak and the strong relative phase between the tree and penguin amplitudes. We can now take  $T$  and  $P$  to be real. For the antiparticle amplitude we have

$$\bar{A}(\bar{B}^0 \rightarrow f_b) = T e^{i(-\delta_w + \delta_s)} + P . \tag{6}$$

Time integrated rate for  $B^0 \bar{B}^0 \rightarrow f_a f_b$  is now given by

$$Rate \sim |\bar{A}(f_a)|^2 \left[ \frac{|A(f_b)|^2 + |\bar{A}(f_b)|^2}{\Gamma^2} + \frac{|A(f_b)|^2 - |\bar{A}(f_b)|^2}{\Gamma^2 + (\Delta m)^2} \right] . \tag{7}$$

The asymmetry is given by

$$\begin{aligned}
A_{sy} &= \frac{\Gamma^2}{\Gamma^2 + (\Delta m)^2} \frac{|A(f_b)|^2 - |\bar{A}(f_b)|^2}{|A(f_b)|^2 + |\bar{A}(f_b)|^2} \\
&= -X_d \frac{2TP \sin \delta_s \sin \delta_w}{T^2 + P^2 + 2TP \cos \delta_s \cos \delta_w}, \tag{8}
\end{aligned}$$

where  $X_d = \Gamma^2/(\Gamma^2 + (\Delta m)^2) \approx 0.5$  [6], which is the dilution factor. It is important to note that the asymmetry arises from the weak phase in the direct amplitude. Thus, in a superweak type model, this asymmetry would be zero. In the following we consider three examples: a)  $f_b = \pi^0 \pi^0$ , b)  $\bar{B}^0 \rightarrow \pi^+ \pi^-$ , and c)  $f_b = \pi^0 K_S$ .

In the SM the decay amplitudes for  $\bar{B}^0 \rightarrow \pi^0 \pi^0$ ,  $\bar{B}^0 \rightarrow \pi^+ \pi^-$ , and  $\bar{B}^0 \rightarrow \pi^0 K_S$  are generated by the following effective Hamiltonian:

$$\begin{aligned}
H_{eff}^q &= \frac{G_F}{\sqrt{2}} [V_{ub} V_{uq}^* (c_1 O_1^q + c_2 O_2^q) - \sum_{i=3}^{10} (V_{ub} V_{uq}^* c_i^u \\
&\quad + V_{cb} V_{cq}^* c_i^c + V_{tb} V_{tq}^* c_i^t) O_i^q] + H.C., \tag{9}
\end{aligned}$$

where the superscript f in  $c_i^f$  indicates the loop contribution from f quark, and  $O_i^q$  are defined as

$$\begin{aligned}
O_1^q &= \bar{q}_\alpha \gamma_\mu L u_\beta \bar{u}_\beta \gamma^\mu L b_\alpha, \quad O_2^q = \bar{q} \gamma_\mu L u \bar{u} \gamma^\mu L b, \\
O_{3,5}^q &= \bar{q} \gamma_\mu L b \bar{q}' \gamma_\mu L(R) q', \quad O_{4,6}^q = \bar{q}_\alpha \gamma_\mu L b_\beta \bar{q}'_\beta \gamma_\mu L(R) q'_\alpha, \\
O_{7,9}^q &= \frac{3}{2} \bar{q} \gamma_\mu L b e_{q'} \bar{q}' \gamma^\mu R(L) q', \quad O_{8,10}^q = \frac{3}{2} \bar{q}_\alpha \gamma_\mu L b_\beta e_{q'} \bar{q}'_\beta \gamma_\mu R(L) q'_\alpha,
\end{aligned} \tag{10}$$

where  $R(L) = 1 + (-)\gamma_5$ , and  $q'$  is summed over u, d, and s. For  $\Delta S = 0$  processes,  $q = d$ , and for  $\Delta S = 1$  processes,  $q = s$ .  $O_2, O_1$  are the tree level and QCD corrected operators.  $O_{3-6}$  are the strong gluon induced penguin operators, and operators  $O_{7-10}$  are due to  $\gamma$  and Z exchange, and “box” diagrams at loop level. The Wilson coefficients  $c_i^f$  are defined at the scale of  $\mu \approx m_b$  which have been evaluated to the next-to-leading order in QCD [7,8]. We give these coefficients below for  $m_t = 176$  GeV,  $\alpha_s(m_Z) = 0.117$ , and  $\mu = m_b = 5$  GeV [8],

$$\begin{aligned}
c_1 &= -0.307, \quad c_2 = 1.147, \quad c_3^t = 0.017, \quad c_4^t = -0.037, \quad c_5^t = 0.010, \quad c_6^t = -0.045, \\
c_7^t &= -1.24 \times 10^{-5}, \quad c_8^t = 3.77 \times 10^{-4}, \quad c_9^t = -0.010, \quad c_{10}^t = 2.06 \times 10^{-3}, \\
c_{3,5}^{u,c} &= -c_{4,6}^{u,c}/3 = P_s^c/3, \quad c_{7,9}^{u,c} = P_e^{u,c}, \quad c_{8,10}^{u,c} = 0
\end{aligned} \tag{11}$$

where  $c_i^t$  are the regularization scheme independent WC's obtained in Ref. [8]. The leading contributions to  $P_{s,e}^i$  are given by:  $P_s^i = (\alpha_s/8\pi)\bar{c}_2(10/9 + G(m_i, \mu, q^2))$  and  $P_e^i = (\alpha_{em}/9\pi)(3\bar{c}_1 + \bar{c}_2)(10/9 + G(m_i, \mu, q^2))$ . The function  $G(m, \mu, q^2)$  is give by

$$G(m, \mu, q^2) = 4 \int_0^1 x(1-x) dx \ln \frac{m^2 - x(1-x)q^2}{\mu^2}. \quad (12)$$

Using the unitarity property of the CKM matrix, we can eliminate the term proportional to  $V_{cb}V_{cq}^*$  in the effective Hamiltonian. The  $B$  decay amplitude due to the complex effective Hamiltonian displayed above can be parametrized, without loss of generality, as

$$< final\ state | H_{eff}^q | B > = V_{ub}V_{uq}^* T_q + V_{tb}V_{tq}^* P_q, \quad (13)$$

where  $T_q$  contains the *tree contributions* and *penguin contributions* due to u and c internal quarks, while  $P_q$  only contains *penguin contributions* from internal c and t quarks.

To obtain exclusive decay amplitudes, we need to calculate relevant hadronic matrix elements. Since no reliable calculational tool exists for two body modes, we shall use factorization approximation to get an idea of the size of asymmetry Asy. The numerical numbers obtained should be viewed as an order of magnitude estimates. The important message is that direct CP violations in decay amplitudes are detectable. Measurements of rate asymmetries at symmetric colliders will provide useful information about CP violation. In the factorization approximation, we have [9]

$$\begin{aligned} T_d(\pi^0\pi^0) &= i \frac{G_F}{\sqrt{2}} f_\pi F_0^{B\pi}(m_\pi^2)(m_B^2 - m_\pi^2)[-c_1 - \xi c_2 + \xi c_3^{cu} + c_4^{cu} \\ &\quad + \frac{3}{2}(c_7^{cu} + \xi c_8^{cu} - c_9^{cu} - \xi c_{10}^{cu}) - \frac{1}{2}(\xi c_9^{cu} + c_{10}^{cu}) \\ &\quad + \frac{2m_\pi^2}{(m_b - m_d)(2m_d)}(\xi c_5^{cu} + c_6^{cu} - \frac{1}{2}(\xi c_7^{cu} + c_8^{cu}))], \\ T_d(\pi^+\pi^-) &= i \frac{G_F}{\sqrt{2}} f_\pi F_0^{B\pi}(m_\pi^2)(m_B^2 - m_\pi^2)[\xi c_1 + c_2 + \xi c_3^{cu} + c_4^{cu} + \xi c_9^{cu} + c_{10}^{cu} \\ &\quad + \frac{2m_\pi^2}{(m_b - m_u)(m_u + m_d)}(\xi c_5^{cu} + c_6^{cu} + \xi c_7^{cu} + c_8^{cu})], \\ T_s(\pi^0\bar{K}^0) &= i \frac{G_F}{\sqrt{2}} \{ f_\pi F_0^{BK}(m_\pi^2)(m_B^2 - m_K^2)[c_1 + \xi c_2 - \frac{3}{2}(c_7^{cu} + \xi c_8^{cu} - c_9^{cu} - \xi c_{10}^{cu})] \\ &\quad - f_K F_0^{B\pi}(m_K^2)(m_B^2 - m_\pi^2)[\xi c_3^{cu} + c_4^{cu} - \frac{1}{2}(\xi c_9^{cu} + c_{10}^{cu})] \} \end{aligned}$$

$$+ \frac{2m_K^2}{(m_b - m_d)(m_d + m_s)} (\xi c_5^{cu} + c_6^{cu} - \frac{1}{2}(\xi c_7^{cu} + c_8^{cu})) \} , \quad (14)$$

where  $c_i^{cu} = c_i^c - c_i^u$ , and  $\xi = 1/N_c$  with  $N_c$  being the number of color. The amplitude  $P_{d,s}$  are obtained by setting  $c_{1,2} = 0$  and changing  $c_i^{cu}$  to  $c_i^{ct} = c_i^c - c_i^t$ . We have used the following decompositions for the form factors

$$\begin{aligned} \langle \pi^+(q) | \bar{d} \gamma_\mu (1 - \gamma_5) u | 0 \rangle &= i f_\pi q_\mu , \quad \langle K^+(q) | \bar{d} \gamma_\mu (1 - \gamma_5) u | 0 \rangle = i f_K q_\mu , \\ \langle \pi^-(k) | \bar{u} \gamma_\mu b | \bar{B}^0(p) \rangle &= (k + p)_\mu F_1^{B\pi} + (m_\pi^2 - m_B^2) \frac{q_\mu}{q^2} (F_1^{B\pi}(q^2) - F_0^{B\pi}(q^2)) , \\ \langle K^-(k) | \bar{u} \gamma_\mu b | \bar{B}^0(p) \rangle &= (k + p)_\mu F_1^{BK} + (m_\pi^2 - m_B^2) \frac{q_\mu}{q^2} (F_1^{BK}(q^2) - F_0^{BK}(q^2)) . \end{aligned} \quad (15)$$

It is a well known fact that in order to obtain asymmetry in rates, the decay amplitudes must contain relative weak and strong rescattering phases. In our case the weak phases are provided by the phase in the CKM matrix elements. For the strong rescattering phases, we use the phases generated at the quark level with the averaged  $q^2 = m_b^2/2$  in eq.(11). The strong phases generated this way are about  $10^\circ$ . The final results for the asymmetries are given in Fig. 1. In the case of  $\bar{B}^0 \rightarrow \pi^0 \pi^0$ , we obtain a large asymmetry if  $\sin \gamma$  is large. The asymmetry can be as large as 12%. The asymmetry for  $\bar{B}^0 \rightarrow \pi^+ \pi^-$  is smaller by a factor about 3. In the case for  $\bar{B}^0 \rightarrow \pi^0 K_S$ , we also obtain smaller asymmetry. In a previous paper we have shown [9] that the rate difference  $\Delta(\pi^0 \pi^0) = \Gamma(\bar{B}^0 \rightarrow \pi^0 \pi^0) - \Gamma(B^0 \rightarrow \pi^0 \pi^0)$  is equal to  $\Delta(\pi^0 K^0) = \Gamma(B^0 \rightarrow \pi^0 K^0) - \Gamma(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0)$  in the SU(3) limit. This gives  $\Delta(\pi^0 \pi^0) = 2\Delta(\pi^0 K_S)$ . One naively expects the asymmetries for both cases to be of the same order. However, this is not the case because the decay amplitude for  $\bar{B}^0 \rightarrow \pi^0 \pi^0$  is dominated by the tree amplitude which is proportional to  $a_2$  and is therefore suppressed, thus increasing the asymmetry, while the decay amplitude for  $\bar{B}^0 \rightarrow \pi^0 K_S$  is dominated by the penguin contributions which is not suppressed, and therefore resulting in a smaller asymmetry [10]. We used two sets of different form factors evaluated in Ref. [12] and Ref. [13]. It is interesting to note that the asymmetries in  $\bar{B}^0 \rightarrow \pi^0 \pi^0$  and  $\bar{B}^0 \rightarrow \pi^+ \pi^-$  are insensitive to the choice of the form factors.

The measurement of asymmetry  $Asy$  can also be used in principle to obtain information

about the weak phase angle  $\gamma$  through the use of eq.(8) with  $\delta_w = \gamma$ . There are likely to be errors in  $T$  and  $P$  evaluation using factorization approximation, but the ratio  $P/T$  is probably more reliable. The largest source of uncertainty is from the evaluation of  $\delta_s$ .  $\delta_s$  calculated at quark level is approximately  $10^\circ$  and this may be good to about 30%. If that is indeed the case,  $\gamma$  will be determined with the same error. Further improvements in the theoretical treatment of nonleptonic B decays are required for a more definitive determination of the weak phase.

We would like to point out that measurements discussed here will also have great impact on the efforts to test the SM by measuring the CKM unitarity triangle. To measure some of the phase angles in the unitarity triangle, it is necessary to measure rate asymmetries in time evolution at asymmetric colliders [4,5,14]. There are two terms varying with time, one varies as a cosine function and the other as a sine function. The coefficient  $C_s$  of the sine term contains information about the phase angles in the unitarity triangle. If the coefficient  $C_c$  (proportional to  $\text{Asy}$ ) of the cosine term is not much smaller than  $C_s$ , like the case for  $\bar{B}^0 \rightarrow \pi^0 \pi^0$ , without knowing the precise value for  $C_c$  the measurement for  $C_s$  will be difficult. Precise measurements of both coefficients  $C_{s,c}$  are required. Although  $C_c$  can also be measured at asymmetric colliders, it is clear that independent measurements of  $C_c$  from a symmetric collider will provide useful information for determining  $C_s$  at an asymmetric collider.

To conclude, we have shown that contrary to the conventional belief time integrated asymmetry are measurable in selected final states in the neutral  $B$  system at symmetric colliders. These asymmetries are indications of direct CP violation and would rule out superweak theories that have CP violation only in  $B^0 - \bar{B}^0$  mass matrix. Our factorization approximation calculation indicates that CP asymmetry in  $\bar{B}^0 \rightarrow \pi^0 \pi^0$  can be as large as 12%. The CP asymmetries in  $\bar{B}^0 \rightarrow \pi^+ \pi^-$  and  $\bar{B}^0 \rightarrow \pi^0 K_S$  are smaller.

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# FIGURES

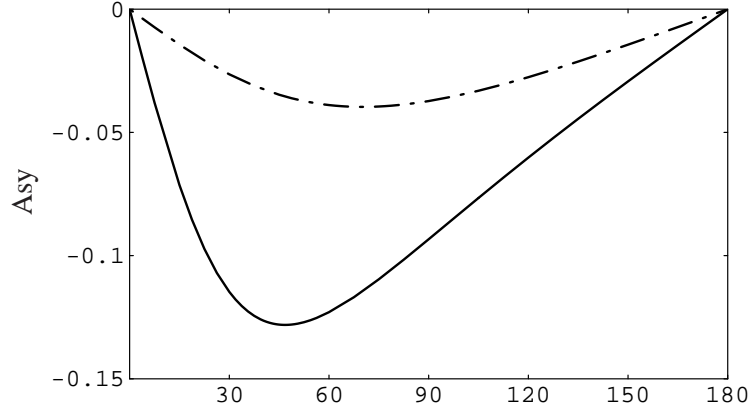


Fig. 1a

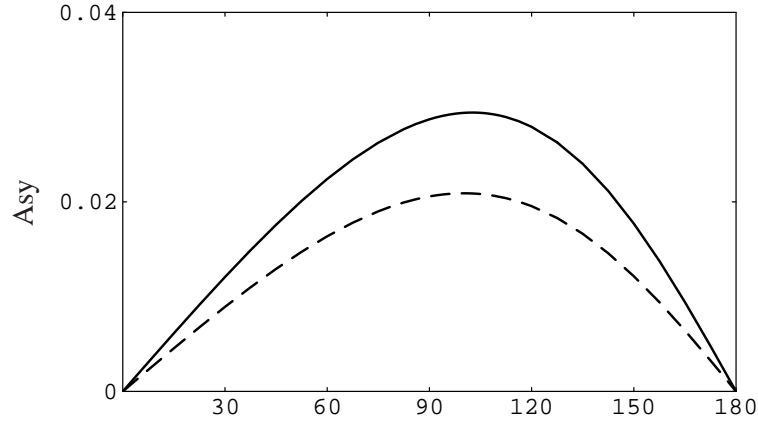


Fig. 1b

FIG. 1. The asymmetry as a function of the phase angle  $\gamma$ . The horizontal axes are  $\gamma$  in degrees. The solid and dot-dashed lines in Fig. 1a are for asymmetries in  $\bar{B}^0 \rightarrow \pi^0\pi^0$  and  $\bar{B}^0 \rightarrow \pi^+\pi^-$ , respectively. The solid and dashed lines in Fig.1b are for asymmetry in  $\bar{B}^0 \rightarrow \pi^0 K_S$  using the form factors in Ref.[11] and Ref.[12], respectively.